RELATIONS & FUNCTIONS

Cartesian Products

$A \times B = \{ (x,y) \mid x \in A \text{ and } y \in B \}$

	1	2	3
x	(1,x)	(2,x)	(3,x)
y	(1,y)	(2,y)	(3,y)

A×B

Set A

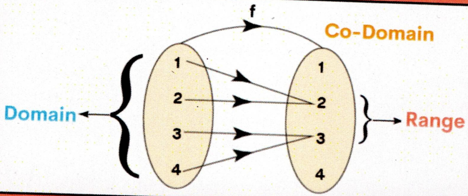
Set B

Properties of Cartesian Products

- $A \times B \neq B \times A$ (non-commutative)
- $A \times \phi = \phi \times A = \phi$
- $n(A \times B) = n(B \times A) = n(A) \times n(B)$
- $n(P(A \times B)) = 2n(A)n(B)$
- If $A \subseteq B$, then $A \times C \subseteq B \times C$
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- $A \times (B - C) = (A \times B) - (A \times C)$
- A and B are two non-empty sets with n elements in common, then $(A \times B)$ & $(B \times A)$ have n^2 element in common.

- **Relations :** A subset of $A \times B$ defined as $(R : A \rightarrow B)$

Total Relations from $A \rightarrow B : 2^{n(A \times B)} = 2^{n(A)} 2^{n(B)}$



Inverse of a relation

- R^{-1}: B → A ⇒ b R a, where a ∈ A, b ∈ B
- R^{-1} = {(b, a) : ∀ (a, b) ∈ R}
  - Domain (R) = Range (R^{-1})
  - Range (R) = Domain (R^{-1})

Classification of Relations, R : A→A={1,2,3}

If R : A→A, Every Relation is a subset of AxA

- **Identity Relation :** I = { (a,a) , a ∈ A }
  - **e.g.** R_1={ (1,1) ; (2,2) ; (3,3) }
- **Reflexive Relation :** (a,a)∈R
  - **e.g.** R_1'={ (1,1) ; (2,2) ; (3,3) ; (1,2) ; (3,2) }
- **Symmetric Relation :** (a,b)∈R_1 ⇒ (b,a)∈R_1 a,b∈A
  - **e.g.** R_2 : { (1,2) ; (2,1) ; (1,1) }
- **Transitive Relation :** (a,b)∈R_2 &(b,c)∈R_2⇒ (a,c)∈R_2
  - **e.g.** R_3 : { (1,2) ; (2,3) ; (1,3) ; (2,2) }

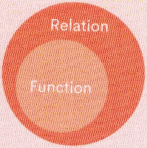
NOTE : Every Identity relation is a reflexive relation but every reflexive relation need not be an Identity.

Equivalence Relation

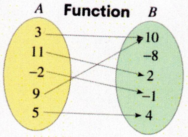
Reflexive | **Symmetric** | **Transitive**

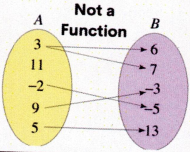
If a relation is all three of above, It is Equivalence

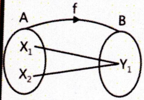
- **Functions :** A relation f:A→B is considered function if
  - Every element of A is associated with some B
  - Association is unique



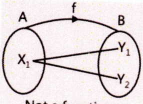
UNIQUE







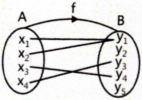
It is a function



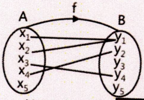
Not a function

One guest from A attending two functions hosted by B

A guest from A is not going to any function hosted by B



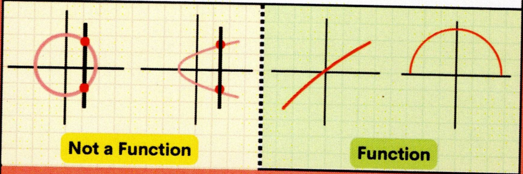
It is a function



Not a function

Vertical Line Method

Draw any line parallel to the y-axis, if it cuts at one point only, then it will be of a function.



Domain of a function (How to find)

- f:A→B All possible input values so that f is defined
- Trick to remember
  - $\dfrac{1}{\text{Expression}}$ Here Expression ≠ 0
  - $\sqrt{\text{Expression}}$, Here Expression ≥ 0

TIPS & TRICKS

- **Example 1 :**

$$f(x) = \frac{1}{x^2 - 8x + 15}$$

$$\Rightarrow f(x) = \frac{1}{(x-5)(x-3)}$$

$$\Rightarrow (x-5)(x-3) \neq 0$$

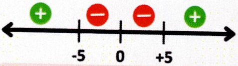
Domain : **R-{5,3}**

- **Example 2 :**

$$f(x) = \sqrt{x^2 - 25}$$

$$\Rightarrow x^2 - 25 \geq 0$$

$$\Rightarrow (x-5)(x+5) \geq 0$$



Domain : **(-∞,-5] U [+5,∞)**

Points Remember about Domain

- $f(x) = 1$; Domain = R
- $f(x) = x/x$; Domain = R-{0}
- In case, two or more expressions are operated, Find individual domains and take intersection
- For a domain of a log function, given as

$log_{f(x)} g(x)$
  - $f(x) \neq 1$; $f(x) > 0$; $g(x) > 0$
  - Take intersection of all expressions

Range of a function (How to find)

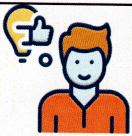
- f:A→B Set of all f-images of elements of A, f(A)
- B is co-domain
- Method to Remember
  - Express x in terms of y
  - Find Possible values for y (Like domain for x)
  - Eliminate values of y with respect to x

$f(x) = \sqrt{16 - x^2}$

Let $y = \sqrt{16 - x^2} \Rightarrow y^2 = 16 - x^2$

$\Rightarrow x^2 = 16 - y^2$

$\Rightarrow x \in [-4,4]$ But $\sqrt{Expression} \geq 0$



Range : **[0,4]**

Equal Functions

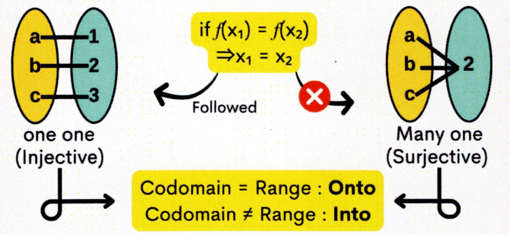
Two functions f & g are said to be equal when

Domain Equal	Co-Domain Equal	$f(x) = g(x)\ \forall x \in$ domain

Types of Functions

Functions, domain, Range and graphs are given in booklet

Classification of Functions



One-One + Onto Function is **Bijective**

Trick to Find One-One or Many One Functions

- **Method 1 : Graphical : Horizontal Line Method**

- **Method 2 : Check Increase or Decrease**
  - If $f(x)$ only increases or decreases, it is one-one
  - If $f(x)$ increases & decreases, it is many-one
- **Method 3 :** if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- **Method 4 :** If $y = f(x)$ is Periodic, Even, Constant , Polynomial of even degree, Non-monotonic, then $f(x)$ is many-One